

Review 2, No Calculator

Complete all the following on notebook paper.

_____ 1.

The point on the curve $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is

- (A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) none of the above

_____ 2.

If $f(x) = \frac{4}{x-1}$ and $g(x) = 2x$, then the solution set of $f(g(x)) = g(f(x))$ is

- (A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1, 2\}$ (E) $\left\{\frac{1}{3}, 2\right\}$

_____ 3.

The region bounded by the x -axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \leq x \leq k$ is three times the area of the region for $k \leq x \leq \frac{\pi}{2}$, then $k =$

- (A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$
- (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

_____ 4.

If the function f is defined by $f(x) = x^5 - 1$, then f^{-1} , the inverse function of f , is defined by $f^{-1}(x) =$

- (A) $\frac{1}{\sqrt[5]{x+1}}$ (B) $\frac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$
- (D) $\sqrt[5]{x}-1$ (E) $\sqrt[5]{x+1}$

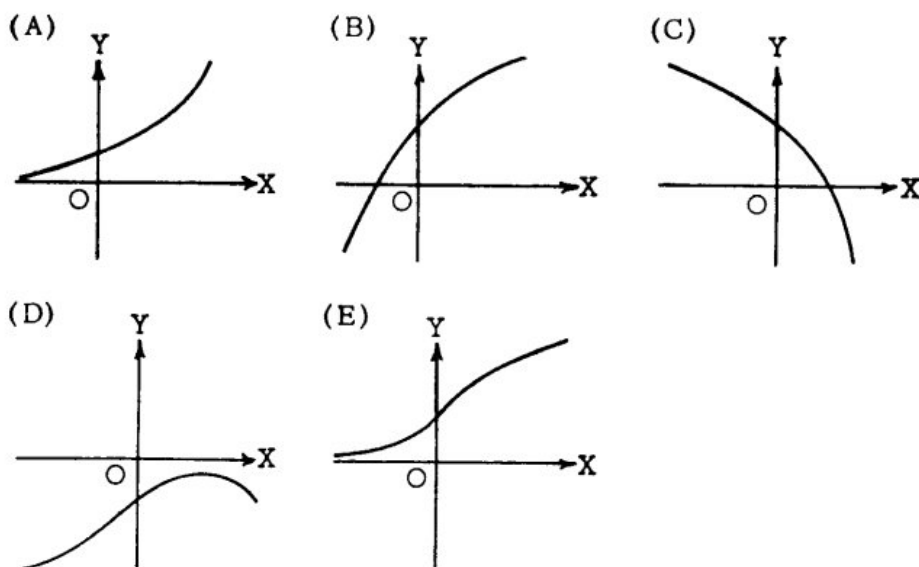
5.

If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x , then the graph of $y = f(x)$ and the graph of $y = g(x)$

- (A) intersect exactly once.
- (B) intersect no more than once.
- (C) do not intersect.
- (D) could intersect more than once.
- (E) have a common tangent at each point of intersection.

6.

If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x , which of the following could be part of the graph of $y = f(x)$?



7.

The graph of $y = 5x^4 - x^5$ has a point of inflection at

- (A) (0,0) only
- (B) (3,162) only
- (C) (4,256) only
- (D) (0,0) and (3,162)
- (E) (0,0) and (4,256)

8.

If $f(x) = 2 + |x - 3|$ for all x , then the value of the derivative $f'(x)$ at $x = 3$ is

- (A) -1
- (B) 0
- (C) 1
- (D) 2
- (E) nonexistent

_____ 9.

A point moves on the x -axis in such a way that its velocity at time t ($t > 0$) is given by $v = \frac{\ln t}{t}$.
At what value of t does v attain its maximum?

- (A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$
(E) There is no maximum value for v .

_____ 10.

An equation for a tangent to the graph of $y = \arcsin \frac{x}{2}$ at the origin is

- (A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$

11. 2000—AB6

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

- (a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.
(b) Find the domain and range of the function f found in part (a).

12. 2001—AB4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

- (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
(b) On what intervals, if any, is the graph of h concave up? Justify your answer.
(c) Write an equation for the line tangent to the graph of h at $x = 4$.
(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?